Taxonomy of the Statistical Characterization of Outage Probability under Indefinite Quadratic Forms

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Abstract

Characterization of key performance indicators (KPIs) such as sum rate, outage probability, bit error rate, and energy efficiency are central in the analysis and design phases of communication networks. These KPIs can be characterized for networks with multiple levels of channel state information (CSI), more commonly, instantaneous CSI and statistical CSI availability is assumed in literature. This paper outlines the need for the statistical CSI based characterization of one such metric, i.e., the outage probability and points out its test cases and provides a brief discussion on the methods and techniques employed to achieve closed-form expression of the KPIs. We outline the recent literature on the use of indefinite quadratic formulation and showcase how the closed-form expression achieved serve as an objective function for the optimization tasks. Use of statistical approaches are also leveraged by non-reliance of feedback paths and hence these methods are considered bandwidth efficient. The paper also discusses that an unbiased quality of service (QoS) for multiple users in the network can be achieved by carefully imposing network constraints. The results indicate that the closed-form expressions are reliable across the tested range.

Keywords: Downlink Communication, Signal Processing, Indefinite Quadratic forms, Statistical CSI, Outage Probability, Sum Rate.

1 Introduction

In the next generation of communication systems, several disruptive technologies are being introduced and thoroughly investigated [1]. A significant area therein is to utilize mmWave and THz spectrum bands [2], and to provide bandwidth efficient solutions for the sub-6 GHz bands [3]. In the later, often literature point towards the use of uplink-downlink channel reciprocity enabled time division multiple access (TDMA) schemes [4] which are based on instantaneous channel state information (CSI) at both ends of a radio access network (RAN), i.e., the base station (BS) and the mobile station (MS). The literature, e.g., [5] points towards making use of instantaneous CSI and then designing the beamforming solutions accordingly. Often the solutions are maximum ratio transmission (MRT), zero-forcing (ZF), regularized ZF, and minimum mean square error (MMSE)

[6]. Depending on the nature of network and constraints imposed therein, a more robust albeit iterative solution is proposed in [7] and it is referred to as branch-reduce-bound (BRB) Algorithm. However, these designs require a feedback path to relay the CSI to transmitter and hence they are not bandwidth efficient since a portion of bandwidth will be used in instantaneous CSI feedback. Hence, for application where a feedback mechanism is non-existent or channel reciprocity is not at play, statistical CSI at the transmitter-based models can be preferred.

Thus, a communication system model wherein the BS has the availability of only the statistical CSI knowledge would not desire feedback path and hence it will save bandwidth. However, modelling such communication systems are mathematically involved as suggested in [8]. Nevertheless, a modern indefinite quadratic forms (IQF) approach [9] which employs complex residue theory, partial fraction expansion, Gaussian integral simplification, and unitary Hermitian matrices structures has enabled the characterization of key performance indicators (KPIs) of conventional communication system. An important point to note herein is that multi-user (MU) system models are considered in recent works alongside multi-antenna BS and/or MS which is natural extension to [9]. These extensions points towards the diversity gains and hence they improve system KPIs. In this paper, we give a review of the available closed-form expressions of a significant KPI, namely, the outage probability (OP) which is developed using the IQF approach. Later we also reflect on the possibility of optimization of such OP by incorporation of transmit and receive beamforming vectors, and we also point towards potential extensions of such models. The frameworks reviewed include multiple input multiple output (MIMO) and multiple input single output (MISO) configurations operating in single and multiple polarization schemes and arbitrary beamformers configurations.

Following the Introduction Section, characterization of OP under several network configurations is given in Section 2. Section 3 points towards the optimization of OP. Section 4 presents validation and optimization results. Conclusion and References sections follows next.

2 Outage Probability Characterization

In simplest terms, OP is a cumulative distributive function (CDF) where we consider that the signal to interference plus noise ratio (SINR) falls below a given threshold, i.e.,

$$OP = \Pr\{SINR < \gamma_{th}\} \tag{1}$$

where γ_{th} is a predefined threshold.

This is a significant metric which is based on SINR and it directly relates with other metrics such as sum rate and spectral efficiency. Presently we have selected works which utilize indefinite quadratic forms herein. More formally, for a given Hermitian matrix **A** and vector **a**, the quadratic form is $\left||\mathbf{a}|\right|_{\boldsymbol{A}}^{\mathbf{2}} = \boldsymbol{a}^H \boldsymbol{A} \boldsymbol{a}$. Here, $(.)^H$ is the conjugate transposition and the matrix **A** can have both positive and negative eigenvalues. The main steps for characterization under this methodology are given in Fig. 1. In what follows, we outline several system models where the OP is characterized using indefinite quadratic formulation.

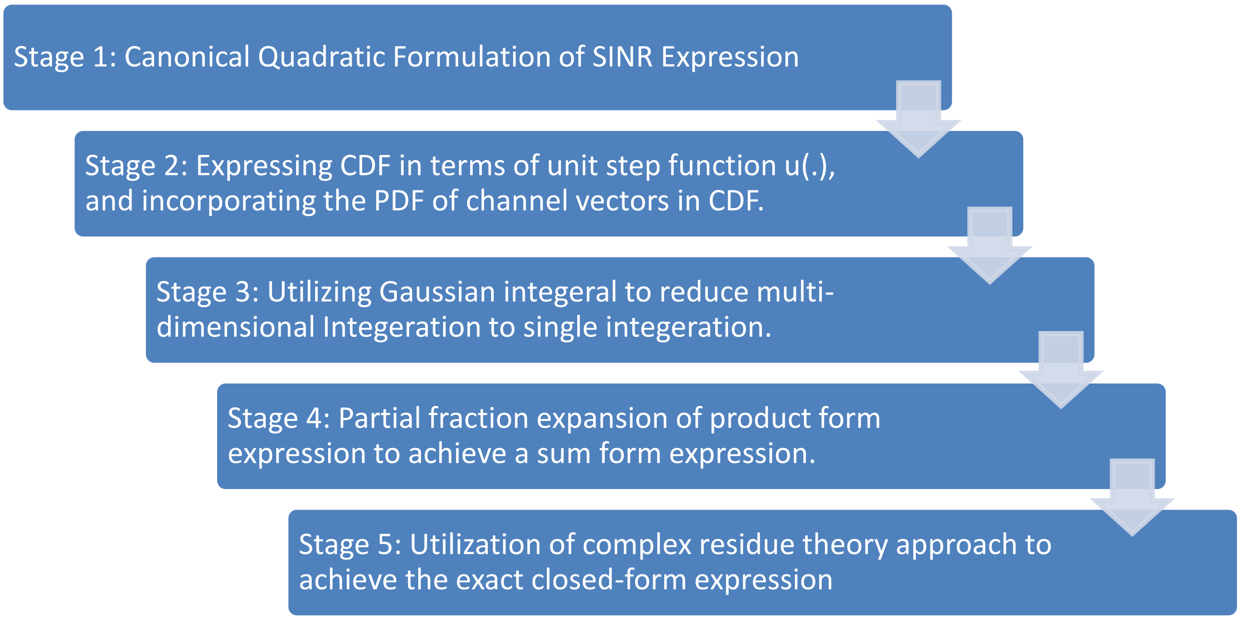

Fig. 1 Five stages in the characterization of OP using Indefinite Quadratic Forms.

2.1: OP characterization of MU-MIMO/MISO systems

Considering Rayleigh fading channels, OP of MU-MISO system is characterized in [10]. The work assumes statical CSI availability at the transmit side and availability of multiple users, the later subjects the desired user to the cochannel interference (CCI) which is a major limiting factor, besides the signal to noise (SNR) power or more specifically the noise power. The closed-form expression of OP for a MU-MISO system characterized in [10] is based on IQF framework originally proposed in [9], and it has the following form:

$$OP = 1 - \sum_i \alpha_i e^{-\beta_i} u(\beta_i) \tag{2}$$

where α and β are functions which are dependent on the beamformers, noise power, and channel correlation information. Furthermore, the closed-form expression is simple since it uses exponential and unit step $u(.)$ functions only.

An extension of the above work is in terms of MU-MIMO system. This consideration is proposed in [11] where both BS and MSs are equipped with multiple antennas. An additional consideration therein is that the self-interference is assumed which incurs higher OP as well. Furthermore, multiple receive antennas are used to achieve equalization gains. The results proposed in that work have the following closed-form expression of OP for the desired user:

$$OP = \sum_j \gamma_j \left(1 - \sum_i \alpha_i e^{-\beta_i} u(\beta_i)\right) \tag{3}$$

where γ is a function obtained using the conditional probability distribution function and it originates from the consideration of self-interference.

2.2: OP characterization under multi-polarized antenna systems.

Closed-form expression of outage probability under the assumption of double or triple polarization while considering statistical CSI availability at the transmit side is provided seminally in [12]. A multi-user system model is considered therein and hence it also accounts for CCI. The closed-form expression has the same form as in (2) albeit the definition of α and β functions were modified, and it catered now for the multi-polarization schemes. The paper showcases that polarization encamped OP expression can by treated as an objective function for a constrained minimization problem as well. The results indicate considerable improvement by such minimization.

2.3: OP characterization under covariance shaping.

The first work of covariance shaping based system modelling was given in [13] which was later extended to [14] and it pointed towards use of equalizers as a tool to shape covariance matrices. For such, Rayleigh quotient-based metrics were used. However, the characterization of OP using IQF while considering the covariance shaping methodology was done in [15]. The paper accounted for joint transmit and receive beamforming using iterative and exhaustive search approaches. The obtained closed-form expression again resembled (2) but the definition of Hermitian weight matrices was now linked with covariance shaping mechanism instead.

2.4: OP characterization for cooperative NOMA networks.

Performance of strong (cell-center) user and weak (cell-edge) user is not similar and for that non-orthogonal multiple access (NOMA) system has recently been showcased as a candidate solution. Therein, superposition coding is performed, and signal power of the strong user and the week user are made dissimilar. At the receiver side, successive interference cancellation (SIC) is performed to decode the message. On these lines, employing IQF and accounting for only the statistical CSI knowledge, the closed-form expression of OP of cell-edge user is given in [15] and it utilizes the unit step function, exponential function, and generalized gamma function. Therein, two solutions are proposed. First deals with linearly linked beamformer vectors and it yields exact closed-form expression of OP, whereas the second considers arbitrary beamformer vectors and this yields approximate expression of OP for the cell-edge user. An extension of this work is given in [17] and that deals with the inclusion of MU-MIMO system rather than MU-MISO systems.

2.5: OP characterization under secure communication constraints.

Leveraging on the use of NOMA schemes discussed above, the assurance of security at the physical layer is modelled and characterized in [18] for MISO NOMA network in a two-user cooperative model. The model includes an eavesdropper which is trying to tap the intended link between legitimate users. Again, using the framework of indefinite quadratic forms, OP of the users and eavesdropper is characterized. The paper also discusses methods of reducing the OP of legitimate users in the presence of eavesdropper by means of optimization criterion.

3 OP Optimization using Beamformers

In the above section, pointers towards OP characterizations for various network configurations are given. The configurations included multiple transmit and/or receive antennas, single and multiple number of users, single and multiple polarization schemes, and possibility of including self-interference in the system model. The characterization of OP enables us to optimize transmit and receive beamformers and hence it can ensure that OP is reduced. However, this optimization is constrained and transmit power shall be restricted to within the permissible range. Furthermore, an unbiased service is needed for all users under consideration. Hence, the optimization problem is casted as follows:

$$\min_{\mathbf{W_{BF}};\boldsymbol{V}_{\mathbf{BF}}} OP(\mathbf{W_{BF}};\boldsymbol{V}_{\mathbf{BF}})$$
$$s.t. \left|\left|\mathbf{W_{BF}}\right|\right|_{\mathbf{2}}^{\mathbf{2}} \leq \mathbf{1}, \forall\, \boldsymbol{users}$$
$$\left|\left|\mathbf{V_{BF}}\right|\right|_{\mathbf{2}}^{\mathbf{2}} \leq \mathbf{1}, \forall\, \boldsymbol{users}$$
$$OP^{optimized} \leq OP^{intialized}, \forall\, \boldsymbol{users}$$

where $\mathbf{W_{BF}}$ and $\mathbf{V_{BF}}$ are the transmit and receive beamformers respectively. The first constraint limits the transmit power, the second constraint enables scaled equalization, and the third constraint is linked with the unbiased criterion where each user benefits from the optimization and gets better results as compared to its initialization state.

The optimization problem presented is non-convex and hence for such, exhaustive search algorithms can be used to find local solution. The literature points to the use of interior-point, active-set, and sequential quadratic programming (SQP) for optimization of such objectives e.g. [15]. Herein, algorithms such as BRB and SDR can provide better and or computationally efficient results, respectively.

4 OP Validation and Discussions

In this section, we showcase the validation setup of the OP expression mentioned in section 2 by considering number of transmit antennas N as 10, number of receive antennas M as 8 and number of users K as 4. Rest of the parameters are as in [10]. In Figure 2, the analytical result is compared with the simulation result, and we observe a good degree of match between the two results thus pointing towards the reliability of indefinite quadratic formulation. Next, in Figure 3, we simplify the model and consider only two users, i.e., K = 2, and perform the optimization outlined in Section 3. In this figure, it is observed that by performing the optimization, the OP of both users has decreased from the initialization point. Also, the improvement is across the set threshold γ value.

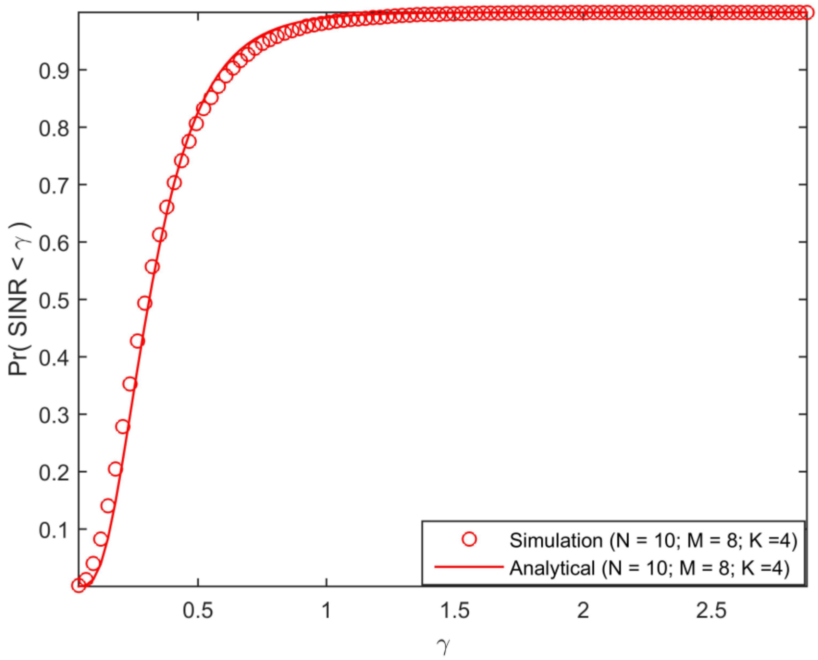

Fig. 2 Closed-form outage probability validation of MU-MIMO system using Monte Carlo simulations.

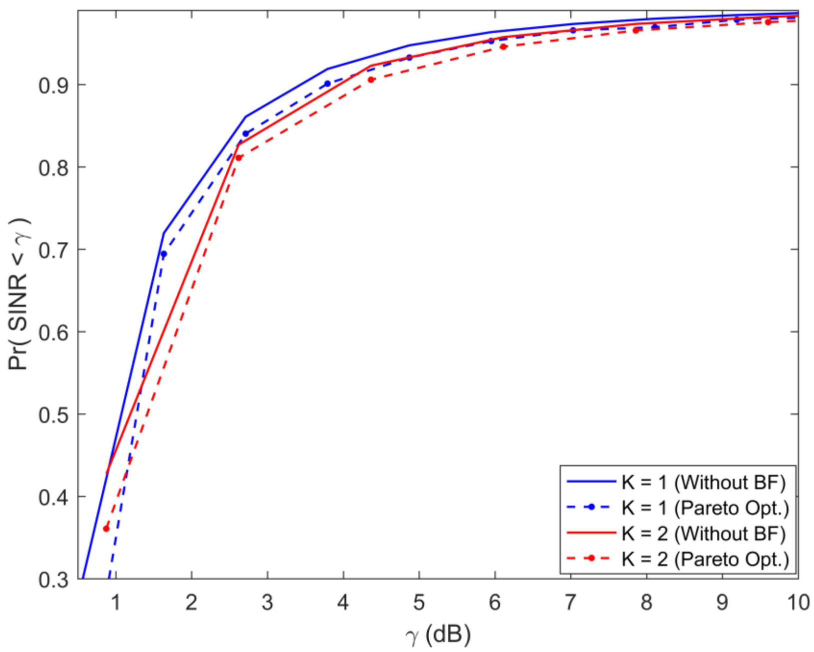

Fig. 3 Optimization of 2-users modeled via indefinite quadratic formulation.

5 Conclusion

This paper outlined recent works on the system modeling and characterization of MU-MIMO and MU-MISO systems under the consideration of statistical CSI availability at the transmit side and Rayleigh fading channels. The brief survey emphasized the validity and test cases for the use of indefinite quadratic formulation in the characterization of outage probability under various network configurations. The paper also highlighted the use of optimization techniques that consider the closed-form expressions of outage probability as an objective function. Optimization is an area that needs further investigation in this domain.

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