A Small Sample Size LCMP Beamformer

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Abstract

In this paper, we consider the problem of automatic diagonal loading for a linearly constrained minimum power beamformer. The proposed method presented in this paper relies on reformulating the linear constrained minimum power problem into a generalized sidelobe canceller formulation to obtain an unconstrained least-squares problem. Then, using the bounded perturbation regularization approach to solve the regularized-least-squares problem. The bounded perturbation regularization method assumes a perturbation matrix with a bounded norm which is added to the linear transformation matrix of the least-squares problem in order to enhance the singular-value structure of the matrix. Compared to different diagonal loading methods, the proposed method shows superiority in performance when small sample size of data is available.

1. Introduction

In many systems, received-spatially-propagating signals may badly be influenced by interference signals. Temporal filtering cannot separate the desired signal from interferers when both occupy the same temporal frequency band. However, usually the desired and interfering signals arise from distinct spatial locations. Hence, this spatial separation can be exploited to isolate the desired signal from interfering signals via a spatial filter. Then, the process that is used in conjunction with an array of antennas, sensors ..., etc., that provide a versatile form of spatial filtering is called *beamforming*; hence, the *beamformer* is the processor that performs spatial filtering when sampling is discrete [1].
Beamforming techniques are common in many communication and satellite systems. They also emerge in a wide range of other applications like RADAR (RAdio Detection And Ranging), SONAR (SOund Navigation And Ranging), imaging, biomedical; etc [1].

Linearly Constrained Minimum Power (LCMP) beamformer constrains the response of the beamformer such that signals from the direction of interest are passed with specified gain and phase. The chosen weights minimize output variance subject to response constraint. This results in preserving the desired signals while eliminating the effect of interference signals and noise arriving from different directions [1], [2].

The second order statistics of the data play a key role in evaluating beamformer performance through estimating the data covariance matrix. Practically, the covariance matrix is unknown and an estimation of it is required. A popular estimator of the covariance matrix is the sample covariance matrix. However, this estimator is unreliable when data is limited [3]. Another practical difficulty is that the receiver does not have accurate spatial characteristic of a specific scenario. This makes filter designing methods rely on assumptions that might not correspond completely to the actual parameters. Several reasons may casue this mismatch which include nonstationarity of the environment, multipath, steering vector errors, etc. [3].

As a result of these challenges, a typical beamformer does not perform well, and Robust Adaptive Beamforming (RAB) techniques are required to mitigate the effect of such mismatches [3]. In the literature, a variety of RAB techniques were proposed.
Interference-plus-Noise Covariance (INC) matrix reconstruction methods aim to reduce the effect of the desired signal by reconstructing the INC [4], [5]. However, the reconstruction process increases the computational complexity. An alternative RAB technique is the uncertainty set based technique which estimates the steering vector of the desired by specifying a spherical uncertainty constraint on the steering vector [6]. However, the performance of this method is limited to low Signal to Noise Ratio (SNR). In addition, this method is computationally inefficient since it requires solving second-order cone programming problems [5].

Steering vector projection is another variation of RAB techniques [7], [8]. The steering vector is replaced by its projection on the signal-plus-interference subspace of the sample covariance matrix, which reduces the effect of noise disturbance. The disadvantages of these methods are that they perform poorly at low SNRs; also, they require perfect knowledge of the dimension of the signal-plus-interference subspace.

Diagonal Loading (DL) is a widely used RAB technique in which the diagonal entries of the sample covariance matrix are altered by a positive value. This technique is also known as regularization in the statistical literature [9]. DL's performance depends on the choice of a scalar loading parameter. Choosing the optimal DL automatically is problematic [3], there is no rigorous way for selecting the parameter since it depends on the noise level [10]. A number of methods were proposed to overcome the problem of automatically choosing the DL parameter. DL methods are efficient if the exact steering vectors of the desired signal and interference signals are known or small mismatches exist.

In this paper, we propose a robust LCMP beamformer based on the bounded perturbation regularization approach [11]. To deal with the constraints in the LCMP beamforming problem, we used the generalized sidlelobe canceller of LCMP that reformulates the problem to an unconstrained least squares problem. The estimated sample covariance matrix which is included in the linear transformation matrix of the LS problem is normally ill-conditioned which makes using regularization approach desirable. The regularization parameter is computed using a procedure that combines a constrained equation with a mean squared error criterion. This allows for automatic adjustment of regularization parameter required by the proposed robust beamformer.

2. LCMP Beamformer with Automatic Diagonal Loading

Typically, the output of a beamformer is obtained from linear combination of the spatially sampled time-series data collected by each element of the array at the input as depicted in Figure (1). The output of a narrowband beamformer is obtained by multiplying a signal $\mathbf{x}(n) \in \mathbb{C}^N$ with a complex weight $\mathbf{w} \in \mathbb{C}^N$ and summing the result to obtain [12]

$$y(n) = \mathbf{w}^H \mathbf{x}(n) \tag{1}$$

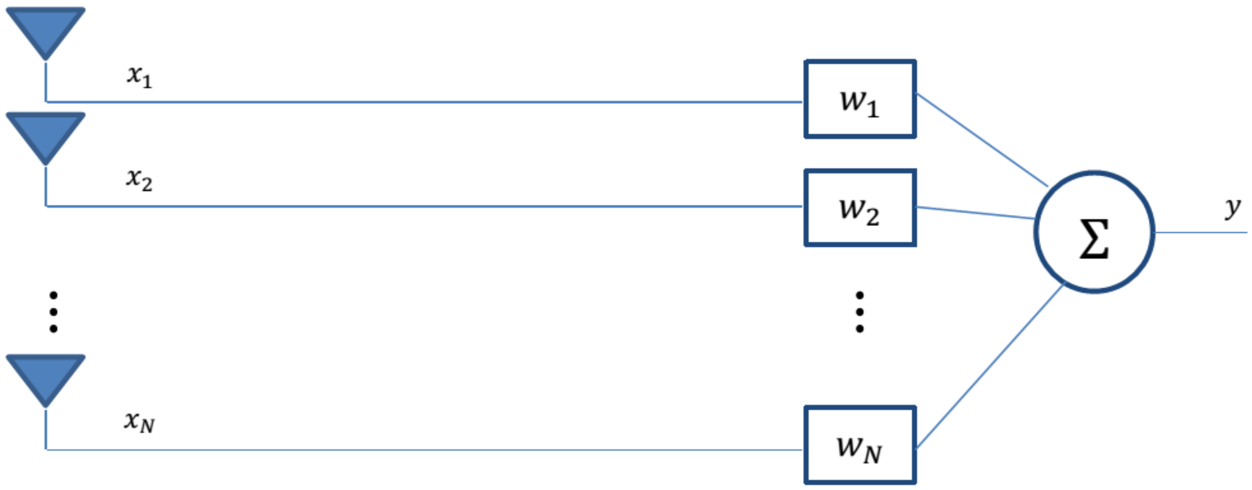


Figure 1. **A beamformer.**

The signal $\mathbf{x}(n)$ constitutes of Q narrowband far-field signals impinging on an array of N elements ($N > Q$), and a vector of Gaussian noise samples, $\mathbf{v}(n) \in \mathbb{C}^N$ is. The n^{th} snapshot of signals received by an array is given by [13]

$$\mathbf{x}(n) = \sum_{i=0}^{Q-1} \mathbf{a}_i s_i(n) + \mathbf{v}(n), \tag{2}$$

where $\mathbf{a}_i$ denotes the steering vector associated with signal i, the subscript i denotes a set of narrowband signals with $i = 0$ (for the desired signal), $i = 1, 2, \ldots, Q-1$ (for interference signals). For the LCMP beamformer, these weights are selected to minimize the output power of the beamformer as follows [12]:

$$\mathbf{w}_{\text{opt}} = \underset{\mathbf{w}}{\operatorname{argmin}} \mathbf{w}^H \mathbf{R}_{\mathbf{x}} \mathbf{w} \quad \text{s.t.} \quad \mathbf{C}^H \mathbf{w} = \mathbf{g}, \tag{3}$$

where $\mathbf{R}_{\mathbf{x}} := \mathbb{E}\left[\mathbf{x}(n)\mathbf{x}^H(n)\right]$ is the data covariance matrix, $\mathbf{C} \in \mathbb{C}^{N \times L}$ is the constraint matrix, and $\mathbf{g}$ is a constraint vector with L elements.

By linearly constraining the weights to satisfy a certain response, $\mathbf{g}$, we ensure that any signal impinging on the array at an angle is passed to the output with the required response.

Using the Lagrange multipliers, the optimum solution of (3) is given by

$$\mathbf{w}_{\text{opt}} = \mathbf{R}_{\mathbf{x}}{}^{-1}\mathbf{C}\left(\mathbf{C}^H \mathbf{R}_{\mathbf{x}}{}^{-1}\mathbf{C}\right)^{-1}\mathbf{g}. \tag{4}$$

Practically, the true covariance matrix is unknown; thus, it is usually replaced by the sample covariance matrix

$$\widehat{\mathbf{R}}_{\mathbf{x}} = \frac{1}{K}\sum_{n=1}^{K} \mathbf{x}(n)\mathbf{x}^H(n), \tag{5}$$

where K is the number of snapshots. The estimated weights of the LCMP beamformer using (5) are given by

$$\mathbf{w}_{\text{lcmp}} = \widehat{\mathbf{R}}_{\mathbf{x}}^{-1}\mathbf{C}\left(\mathbf{C}^H \widehat{\mathbf{R}}_{\mathbf{x}}^{-1}\mathbf{C}\right)^{-1}\mathbf{g} \tag{6}$$

The generalized sidelobe canceller provides an alternative implementation of the LCMP beamformer. Basically, it changes the constrained minimization problem introduced in (3) into unconstrained form by decomposing $\mathbf{w}$ into two components; the first one is in the constraint subspace, and the second one is orthogonal to the first [12], i.e.,

$$\mathbf{w} = \mathbf{w}_q - \mathbf{B}\mathbf{w}_a \tag{7}$$

where $\mathbf{w}_q \in \mathbb{C}^N$ is a fixed quiescent weight vector and $\mathbf{B} \in \mathbb{C}^{N \times (N-L)}$ is a blocking matrix.
By substituting (7) in (3) and replacing $\mathbf{R}_\mathbf{x}$ with $\widehat{\mathbf{R}}_\mathbf{x}$, the problem can be reformulated as the following unconstrained LS:

$$\min_{\mathbf{w}_a} \|\mathbf{A}\mathbf{w}_a - \mathbf{b}\|^2, \tag{8}$$

where $\mathbf{A} := \widehat{\mathbf{R}}_\mathbf{x}^{-\frac{1}{2}}\mathbf{B} \in \mathbb{C}^{N \times (N-L)}$, and $\mathbf{b} := \widehat{\mathbf{R}}_\mathbf{x}^{-\frac{1}{2}}\mathbf{w}_q$. The above minimization can be shown corresponds to a linear regression model with noise, which is a suitable model for applying regularization to estimate $\mathbf{w}_a$. The Regularized Least Squares (RLS) problem reads as follows:

$$\min_{\mathbf{w}_a} \|\mathbf{A}\mathbf{w}_a - \mathbf{b}\|^2 + \gamma\|\mathbf{w}_a\|^2, \tag{9}$$

which has the closed-form solution

$$\widehat{\mathbf{w}}_a = (\mathbf{A}^H\mathbf{A} + \gamma\mathbf{I})^{-1}\mathbf{A}^H\mathbf{b}. \tag{10}$$

It is remarkable from (10) that we consider regularizing $\mathbf{A}^H\mathbf{A}$, which is of dimension $(N-L)\times(N-L)$ instead of $\widehat{\mathbf{R}}_\mathbf{x}$ which is of an $N \times N$ dimension. Hence, the inversion (10) is valid for fewer snapshots.
The proposed selection method of the regularization parameter is based on minimizing the mean squared error in estimating $\mathbf{w}_a$, i.e,

$$\text{MSE} = \operatorname{tr}[\mathbb{E}[(\mathbf{w}_a - \widehat{\mathbf{w}}_a)(\mathbf{w}_a - \widehat{\mathbf{w}}_a)^H]], \tag{11}$$

where tr (.) denotes the matrix trace.

A detailed procedure following this step can be found in [11] and [14]. At the end, we automatically select the regularization parameter (diagonal loading) as a root for the following Bounded Perturbation Regularization (BPR) equation:

$$f(\gamma) = \operatorname{tr}[(\mathbf{\Sigma}^2 + \gamma\mathbf{I})^{-1}]\operatorname{tr}[(\mathbf{\Sigma}^2 + \gamma\mathbf{I})^{-1}\mathbf{d}\mathbf{d}^H] - N[(\mathbf{\Sigma}^2 + \gamma\mathbf{I})^{-2}\mathbf{r}\mathbf{r}^H] = 0, \tag{12}$$

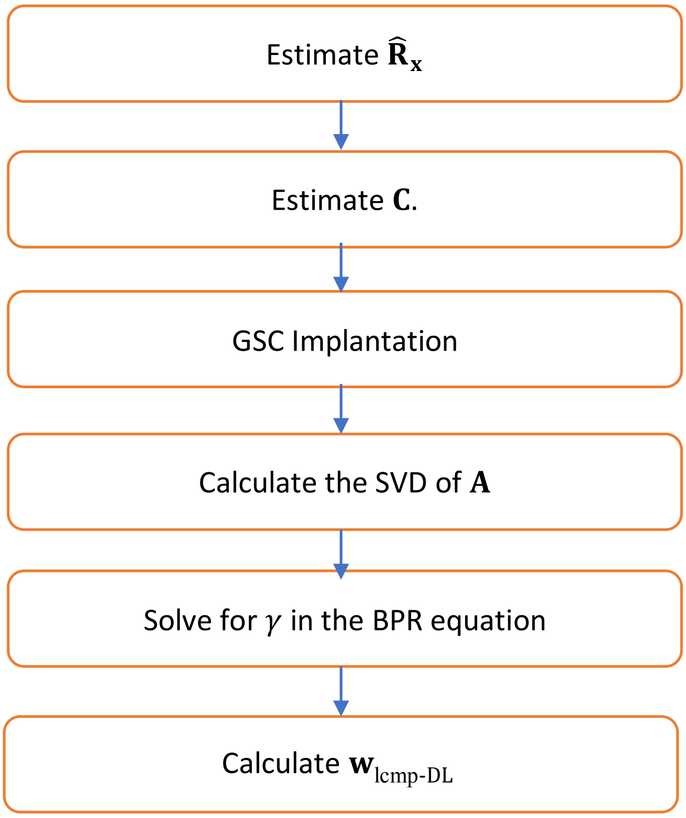
where we use the Singular Value Decomposition (SVD) of $\mathbf{A} = \mathbf{U\Sigma V}^H$, and $\mathbf{r} := \mathbf{U}^H\mathbf{b}$.
Finally, we substitute the regularization parameter, γ , obtained from (12) in the loaded version of (6), to obtain the following:

$$\mathbf{w}_{\text{lcmp-DL}} = (\widehat{\mathbf{R}}_\mathbf{x} + \gamma\mathbf{I})^{-1}\mathbf{C}\left(\mathbf{C}^H(\widehat{\mathbf{R}}_\mathbf{x} + \gamma\mathbf{I})^{-1}\mathbf{C}\right)^{-1}\mathbf{g} \tag{13}$$

3. Summary of the Method

The following steps summarize the method:

- Step 1: From the received data, $\mathbf{x}(n)$, estimate $\widehat{\mathbf{R}}_{\mathbf{x}}$ according to Equation (5).
- Step 2: Estimate the steering vectors of the desired and the interference signals: This is to composite matrix, **C**.
- Step 3: Use the GSC implementation: This requires calculating matrix A which is defined just after Equation (8). Calculating A requires estimating the block matrix B which is an arbitrary matrix chosen to be orthogonal to the constraint matrix (i.e., $\mathbf{B}^H\mathbf{C} = \mathbf{0}$).
- Step 4: Do the singular value decomposition (SVD) of **A**.
- Step 5: Use Newton's method to solve the BPR equation (12).
- Step 7: Calculate the weights according to Equation (13).



4. Performance Evaluation

To evaluate performance, the output signal-to-interference and-noise-ratio (SINR) is considered. Equation (2) can be written as $\mathbf{x}(n) = \mathbf{x}_s(n) + \mathbf{x}_{iv}(n)$, where $\mathbf{x}_s(n) := \mathbf{a}_0 s_0(n)$, and $\mathbf{x}_{iv}(n) := \sum_{i=1}^{Q-1} \mathbf{a}_i s_i(n) + \mathbf{v}(n)$. The output SINR is defined as follows:

$$\mathrm{SINR} = \frac{\mathbb{E}\,[|\mathbf{w}^H \mathbf{x}_s(n)|^2]}{\mathbb{E}\,[|\mathbf{w}^H \mathbf{x}_{iv}(n)|^2]},$$

$$= \frac{\sigma_s^2 |\mathbf{w}^H \mathbf{a}_{0t}|^2}{\mathbf{w}^H \mathbf{R}_{i+n} \mathbf{w}}, \tag{14}$$

where $\mathbf{R}_{i+n} := \mathbb{E}\,[\mathbf{x}_{iv}(n)\mathbf{x}_{iv}{}^H(n)]$ is the INC matrix, σ_s^2 is the power of the signal of interest, and $\mathbf{a}_{0t}$ is the actual steering vector of the desired signal.

In all scenarios, we compare the proposed LCMP-BPR with HKB [15], [16], elliptical regularized sample covariance matrix (ELL-RSCM) [17], GLC [10], multichannel wiener filtering based noise reduction with truncated minimum mean square error criterion (MWF-TMMSE) [13], and tridiagonal loading (TRI) [18] methods. Similar to the proposed method, HKB and TRI methods are one parameter diagonal loading methods. The other methods, Ell-RSCP, GLC and MWFTMMSE, estimate the covariance matrix via a shrinkage method that uses two regularization parameters.

A uniform linear array (ULA) of $N = 10$ elements with $d = 0.5\lambda$ between consecutive elements is used in all simulations, where λ is the wavelength. Uncertainty in the direction of arrival (DOA) of the signal of interest is modeled as a uniform distribution in the range $[-2^\circ, 2^\circ]$. We consider six interference signals ($Q = 7$) impinging on the array.

The signal of interest and interference signals are complex Gaussian data generated randomly with SNR = 5 dB and Interference-to-Noise Ratio (INR = 20 dB). The noise is complex white Gaussian with unit-norm power. Source locations are randomly chosen in every iteration. All SINR curves are obtained by averaging over 2×10^4 independent trials. For this LCMP beamformer, three signals out of the six interference signals are constrained to nulls, i.e., the constraint vector is $\mathbf{g} = [1, 0, 0\ ,0]^T$, (i.e., $L = 4$)

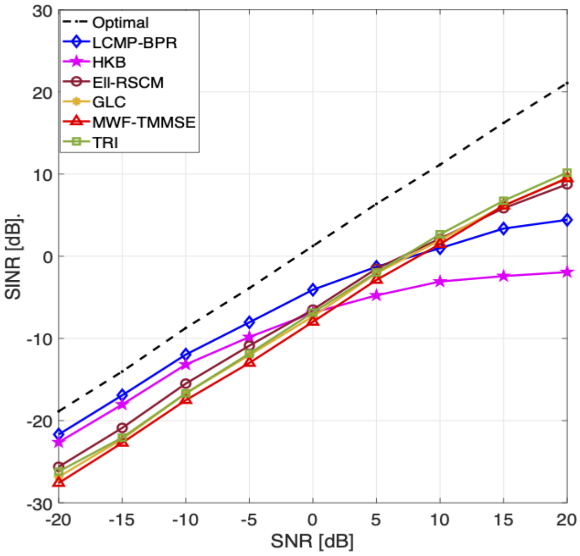


Figure 3 **SINR vs. SNR (*K* = 7)**

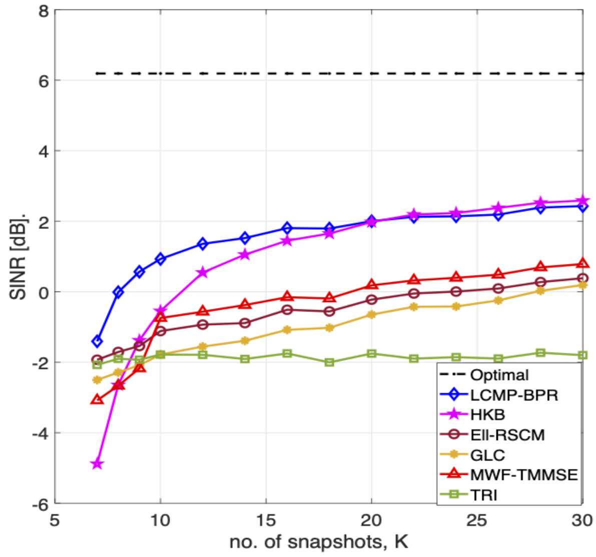


Figure 2 **SINR vs. K (SNR = 5 dB).**

Figure 2 shows the output SINR versus SNR. As can be seen from the figure, for SNR ≤ 10 dB, the proposed LCMP-BPR method achieves the best performance among all the methods. For 10 dB < SNR ≤ 20 dB, LCMP-BPR exhibits an inferior performance compared to the other methods except HKB.

Figure 3 shows the output SINR versus the number of snapshots, K. As can be seen, when N − P < K < 2N, LCMP-BPR outperforms all the other techniques. However, for 2N ≤ K ≤ 3N, HKB shows a slight advantage over the proposed LCMP-BPR

5. Conclusion

We propose the LCMP-BPR beamformer based on the bounded perturbation regularization approach. A generalized sidelobe canceller implementation of a linearly constrained minimum power was considered. The constrained LCMP problem was reformulated into an unconstrained least squares problem. Simulation results show that the proposed LCMP-BPR method is effective in scenarios with a ULA of N elements, P constraints, and limited number of snapshot K ∈ (N − P, 3N].

6. References

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